

Engineering Waths First Aid Kit

Quadratic equations 1

Introduction

This leaflet will explain how many quadratic equations can be solved by **factorisation**.

1. Quadratic equations

A quadratic equation is an equation of the form $ax^2 + bx + c = 0$, where a, b and c are constants. For example $3x^2 + 2x - 9 = 0$ is a quadratic equation with a = 3, b = 2 and c = -9.

The constants b and c can have any value including 0. The constant a can have any value except 0. This is to ensure that the equation has an x^2 term. We often refer to a as the coefficient of x^2 , to b as the coefficient of x and to c as the constant term. Usually, a, b and c are known numbers, whilst x represents an unknown quantity which we will be trying to find.

2. The solutions of a quadratic equation

To **solve** a quadratic equation we must find values for x which when substituted into the equation make the left-hand and right-hand sides equal. These values are also called **roots**. For example the value x = 4 is a solution of the equation $x^2 - 3x - 4 = 0$ because substituting 4 for x we find

 $4^2 - 3(4) - 4 = 16 - 12 - 4$

which simplifies to zero, the same as the right-hand side of the equation. There are several techniques which can be used to solve quadratic equations. One of these, *factorisation*, is discussed in this leaflet. You should be aware that not all quadratic equations can be solved by this method. An alternative method which uses a formula is described on leaflet 2.15.

3. Solving a quadratic equation by factorisation.

Sometimes, but not always, it is possible to solve a quadratic equation using factorisation. If you need to revise factorisation you should see leaflet 2.6 Factorising quadratics.

Example

Solve the equation $x^2 + 7x + 12 = 0$ by factorisation.

Solution

We first factorise $x^2 + 7x + 12$ as (x+3)(x+4). Then the equation becomes (x+3)(x+4) = 0.

It is important that you realise that if the product of two quantities is zero, then one or both of the quantities must be zero. It follows that either

x + 3 = 0, that is x = -3 or x + 4 = 0, that is x = -4

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The roots of $x^2 + 7x + 12 = 0$ are x = -3 and x = -4.

Example

Solve the quadratic equation $x^2 + 4x - 21 = 0$.

Solution

 $x^{2} + 4x - 21$ can be factorised as (x + 7)(x - 3). Then

$$x^{2} + 4x - 21 = 0$$

(x + 7)(x - 3) = 0

Then either

x + 7 = 0, that is x = -7 or x - 3 = 0, that is x = 3

The root of $x^2 + 4x - 21 = 0$ are x = -7 and x = 3.

Example

Find the roots of the quadratic equation $x^2 - 10x + 25 = 0$.

Solution

$$x^{2} - 10 + 25 = (x - 5)(x - 5) = (x - 5)^{2}$$

Then

$$x^{2} - 10x + 25 = 0$$

(x - 5)² = 0
x = 5

There is one root, x = 5. Such a root is called a **repeated root**.

Example

Solve the quadratic equation $2x^2 + 3x - 2 = 0$.

Solution

The equation is factorised to give

$$(2x - 1)(x + 2) = 0$$

so, from 2x - 1 = 0 we find 2x = 1, that is $x = \frac{1}{2}$. From x + 2 = 0 we find x = -2. The two solutions are therefore $x = \frac{1}{2}$ and x = -2.

Exercises

1. Solve the following quadratic equations by factorization.

- a) $x^2 + 7x + 6 = 0$, b) $x^2 8x + 15 = 0$, c) $x^2 9x + 14 = 0$,
- d) $2x^2 5x 3 = 0$, e) $6x^2 11x 10 = 0$, f) $6x^2 + 13x + 6 = 0$.

Answers

a) -1, -6, b) 3, 5, c) 2, 7, d) 3, $-\frac{1}{2}$, e) $\frac{5}{2}, -\frac{2}{3},$ f) $x = -\frac{3}{2}, x = -\frac{2}{3}.$

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