# Quadratic equations 1 

## Introduction

This leaflet will explain how many quadratic equations can be solved by factorisation.

## 1. Quadratic equations

A quadratic equation is an equation of the form $a x^{2}+b x+c=0$, where $a, b$ and $c$ are constants. For example $3 x^{2}+2 x-9=0$ is a quadratic equation with $a=3, b=2$ and $c=-9$.
The constants $b$ and $c$ can have any value including 0 . The constant $a$ can have any value except 0 . This is to ensure that the equation has an $x^{2}$ term. We often refer to $a$ as the coefficient of $x^{2}$, to $b$ as the coefficient of $x$ and to $c$ as the constant term. Usually, $a, b$ and $c$ are known numbers, whilst $x$ represents an unknown quantity which we will be trying to find.

## 2. The solutions of a quadratic equation

To solve a quadratic equation we must find values for $x$ which when substituted into the equation make the left-hand and right-hand sides equal. These values are also called roots. For example the value $x=4$ is a solution of the equation $x^{2}-3 x-4=0$ because substituting 4 for $x$ we find

$$
4^{2}-3(4)-4=16-12-4
$$

which simplifies to zero, the same as the right-hand side of the equation. There are several techniques which can be used to solve quadratic equations. One of these, factorisation, is discussed in this leaflet. You should be aware that not all quadratic equations can be solved by this method. An alternative method which uses a formula is described on leaflet 2.15.

## 3. Solving a quadratic equation by factorisation.

Sometimes, but not always, it is possible to solve a quadratic equation using factorisation. If you need to revise factorisation you should see leaflet 2.6 Factorising quadratics.

## Example

Solve the equation $x^{2}+7 x+12=0$ by factorisation.

## Solution

We first factorise $x^{2}+7 x+12$ as $(x+3)(x+4)$. Then the equation becomes $(x+3)(x+4)=0$.
It is important that you realise that if the product of two quantities is zero, then one or both of the quantities must be zero. It follows that either

$$
x+3=0, \text { that is } x=-3 \quad \text { or } \quad x+4=0, \text { that is } x=-4
$$

The roots of $x^{2}+7 x+12=0$ are $x=-3$ and $x=-4$.

## Example

Solve the quadratic equation $x^{2}+4 x-21=0$.

## Solution

$x^{2}+4 x-21$ can be factorised as $(x+7)(x-3)$. Then

$$
\begin{array}{r}
x^{2}+4 x-21=0 \\
(x+7)(x-3)=0
\end{array}
$$

Then either

$$
x+7=0, \text { that is } x=-7 \quad \text { or } \quad x-3=0, \text { that is } x=3
$$

The root of $x^{2}+4 x-21=0$ are $x=-7$ and $x=3$.

## Example

Find the roots of the quadratic equation $x^{2}-10 x+25=0$.

## Solution

$$
x^{2}-10+25=(x-5)(x-5)=(x-5)^{2}
$$

Then

$$
\begin{aligned}
x^{2}-10 x+25 & =0 \\
(x-5)^{2} & =0 \\
x & =5
\end{aligned}
$$

There is one root, $x=5$. Such a root is called a repeated root.

## Example

Solve the quadratic equation $2 x^{2}+3 x-2=0$.

## Solution

The equation is factorised to give

$$
(2 x-1)(x+2)=0
$$

so, from $2 x-1=0$ we find $2 x=1$, that is $x=\frac{1}{2}$. From $x+2=0$ we find $x=-2$. The two solutions are therefore $x=\frac{1}{2}$ and $x=-2$.

## Exercises

1. Solve the following quadratic equations by factorization.
a) $x^{2}+7 x+6=0$,
b) $x^{2}-8 x+15=0$,
c) $x^{2}-9 x+14=0$,
d) $2 x^{2}-5 x-3=0$,
e) $6 x^{2}-11 x-10=0$,
f) $6 x^{2}+13 x+6=0$.

## Answers

a) $-1,-6$,
b) 3,5 ,
c) 2,7 ,
d) $3,-\frac{1}{2}$,
e) $\frac{5}{2},-\frac{2}{3}$,
f) $x=-\frac{3}{2}, x=-\frac{2}{3}$.

